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Exploiting Trust Relations for Nash Equilibrium Efficiency in Ad Hoc Networks

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Abstract—**Ad hoc networks rely on the mutual cooperation among individual nodes to achieve network-wide objectives. However, individual nodes may behave selfishly in order to maximize their own benefits without considering the global benefits of the network. One approach to incentivize nodes cooperation for better global benefits is to establish trust relations among nodes to guide their decision making. In this paper, we present a game theoretic analysis for the efficiency of establishing trust for improving node cooperation. The trust relations among nodes are modeled as a trust-weighted network, and we study a graphical game in this network where the nodes' payoffs are affected by their trust relations. We characterize the Nash equilibrium and the social optimum of this game and show that the game efficiency has a close relationship to the Bonacich centralities of nodes in the trust-weighted network. Furthermore, we propose to improve game efficiency by introducing heterogeneous resources to nodes according to their centralities. We provide both experimental and theoretical analysis on the improvement of the game efficiency.**

I. Introduction

Ad hoc networks rely on the mutual cooperation among nodes to achieve network-wide goals. However, due to resource constraints, these nodes may not be willing to cooperate as cooperation consumes their own resources. A large amount of research have been devoted to studying the collective behaviors of such selfish nodes [1]–[3]. By modeling the interactions among nodes as a game, the Nash equilibria are used to study the operating points of the network. It's shown that such operating points usually have inferior performance when compared to the social optimum, which is the operating point that all nodes are subject to centralized control [3]. To encourage cooperation from the self-interested nodes in the Nash equilibria, various incentive mechanisms have been proposed in literature [4]–[6]. One promising technique is to establish trust relations among nodes. Most literature work focus on the design of different aspects of the trust schemes, such as trust evaluation, trust information storage and retrieval. However, few effort has been made on quantitatively analyzing the efficiency of establishing trust for improving node cooperation. In this paper, we provide a game theoretic analysis as a first step towards this end.

We model the trust relations among nodes as a trust-weighted network and study a simple game in which the user's payoff depends on the contributions from itself and its neighbors, weighted by the trust values associated with each neighbor. This game can capture the efforts of nodes to coordinate across the network and the effects of trust relations on nodes' efforts. We analyze the characteristics of the Nash equilibrium and social optimum of this game and show that they have a close relationship to the Bonacich centralities of nodes in the trust-weighted network. We then evaluate the efficiency of the game which measures the effect of establishing trust for improving node cooperation. The efficiency of the game is measured by the price of anarchy [2], which is defined to be the ratio of the payoff at the worst Nash equilibrium to that of the social optimum. It's demonstrated that the node centralities in the trust-weighted network are very important for determining the game efficiency. Moreover, motivated by the observation that nodes with heterogeneous resources have different willingness to cooperate, we exploit the potential of improving game efficiency by introducing heterogeneous resources. Critical issues on this aspect include where and how many resources to introduce. We provide both experimental and mathematical analysis and show that the resources should be introduced according to node centralities in the trust-weighted network.

The rest of the paper is organized as follows. Section II introduces the related work. In Section III, we describe the game theoretical formulation. Section IV is devoted to characterization of the Nash equilibrium and the social optimum. In Section V, we propose how to exploit the structure of the trust-weighted network to improve game efficiency. Section VI gives the conclusions and the future work.

II. Related Work

Most literature work on establishing trust relations in ad hoc network have focused on the design of the trust models. For example, Seredynski et at. [4] proposed a trust model for the routing service in ad hoc networks. Their model encourages cooperation among nodes through the incentives they receive: more cooperation gives a node higher trust which results in better service for itself. Baras and Jiang [5] proposed a model based on cooperative games, in which the nodes can form coalitions in order to maximize their payoffs. Their scheme utilizes a feedback relation among the strategy, the payoff and the trust level. More work on game theoretic model on trust and cooperation can be found in [7]–[9]. While the solutions put forth in these references address important issues, most of them did not consider the relation between game efficiency and the structure of the trust relations among nodes. In this paper, we propose an initial analysis on this aspect.

Our work is partly motivated by the work in [10], which investigated the effect of network topology in a model of content contribution in peer-to-peer networks with linear quadratic payoffs. They identified the relation between Bonacich cen-

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tralities and Nash equilibria within their game model and discussed a network-based policy for improving the equilibrium performance by exclusion of a single player. However, no trust relations are considered in their model, and the effect of network topology on game efficiency is not analyzed. In our work, we use a more general payoff function, and focus the analysis on game efficiency and trust relations among nodes.

III. Problem Formulation

To investigate the effect of establishing trust for improving node cooperation in Nash equilibria, we study a game in which the network participants can experience a marginal increase in payoff from neighbors' contributions. This type of relations among neighbors are very common in practice. For example, in a peer-to-peer file sharing system, users can benefit from the content provided by neighbors; in a network under malicious attacks, the security technologies a user deploys to protect itself will also benefit its neighbors.

We model the trust relations among nodes as a directed graph $G = \{V, E\}$ associated with a trust matrix $T = [t_{ij}]$, where $t_{ij} \in [0, 1]$ represents the trust opinion from i to j. Due to the asymmetric property of trust relations, t_{ij} may not be equal to t_{ji}. The set of i's neighbors, denoted by $\mathcal{N}_i$, is the set of users that have trust relations with i, i.e., $\mathcal{N}_i = \{j \in V | t_{ij} \neq 0\}$. The payoff of a user depends on the contributions from itself and its neighbors. Each user simultaneously chooses a contribution level $x_i \geq 0$. The effect of neighbors' contribution on the node is captured by a parameter $\delta \in (0, 1]$. Since nodes are connected by their trust relations, the actual contributions received by node i are weighted by the trust values associated with each neighbor. In summary, the contribution that node i receives from its neighbors is $\delta \sum_{j\in\mathcal{N}_i} t_{ij}x_j$. We assume that each user receives benefits according to a benefit function $b(x_i + \delta \sum_{j\in\mathcal{N}_i} t_{ij}x_j)$. Assuming that the cost for making unit contribution is c, then the benefit function satisfies the conditions that $b(0) = 0$, $b'(0) > c$ and $b'(+\infty) < c$, as no user will make any effort if $b'(0) < c$ and all users will make infinite efforts if $b'(+\infty) > c$. The users are rational and risk averse, so the benefit function $b(\cdot)$ is concave [11]. Denote the profile of all users other than i by $\mathbf{x}_{-i}$, then the payoff for user i is

$$u_i(x_i, \mathbf{x}_{-i}) = b(x_i + \delta \sum_{j\in\mathcal{N}_i} t_{ij}x_j) - c \cdot x_i, \quad (1)$$

where $x_i \in [0, +\infty)$. Actually, since $b'(0) > c$ and $b'(+\infty) < c$, there exists some $w > 0$ such that $b'(w) = c$, then the range of effort x_i becomes $[0, w]$.

This simple form of payoff can capture the efforts of nodes to coordinate across the network and the effects of trust relations on nodes' efforts. Moreover, it allows us to focus on the structure of the trust-weighted network. To better illustrate the payoff definition in equation (1), we give the following example of a p2p file sharing system. Suppose user i shares x_i files with neighbors, and with probability t_{ij} it expects its neighbor j to provide good content rather than malicious one, then the expected total number of good files that i can reach is $F_i = x_i + \sum_{j\in\mathcal{N}_i} t_{ij}x_j$. The benefit user i obtains is a concave function of F_i, say $b(F_i)$; then if the cost for providing files is linear, the payoff of user i can be represented by $b(x_i + \sum_{j\in\mathcal{N}_i} t_{ij}x_j) - c \cdot x_i$, which is the case of equation (1) for $\delta = 1$.

IV. Nash Equilibria and Efficiency Analysis

Before we introduce the analysis of the Nash equilibria for this game, we first introduce a network centrality measure put forth by Bonacich [12]. Given a scalar α and a network with adjacency matrix A such that $(I - \alpha A)$ is invertible, the vector of *Bonacich centralities* for the nodes is defined to be $\mathbf{h}(\alpha, A) = (I - \alpha A)^{-1} A \cdot \mathbf{1}$, where $\mathbf{1}$ is the vector of all 1. Let $a_{ij}^{(k)}$ be the element in the i^{th} row and j^{th} column of matrix A^k, the kth power of A, and let $\rho(A)$ be the spectral radius of A, i.e., the largest absolute value of A's eigenvalues , then if $|\alpha| < \rho(A)$, we have $h_i(\alpha, A) = \sum_{k=0}^{\infty} \alpha^k \sum_j a_{ij}^{(k+1)}$, where $h_i(\alpha, A)$ is the Bonacich centrality for node i. We can see that $a_{ij}^{(k)}$ actually accounts for the total weight of all paths of length k from user i to user j and α acts as a decay factor that scales down the relative weight of longer paths. If $\alpha < 0$, $h_i(\alpha, A)$ puts positive weights on node i's neighbors, but negative weights on the neighbors' neighbors, and so on. In other words, the larger weights the node's neighbors put on their other neighbors, the less central is the node.

We now analyze the Nash equilibria of the game defined in Section III. Due to the concavity of the benefit function and linearity of cost, there are no mixed equilibria for this game. More specifically, a user would always get a higher payoff by playing the average of a set of contribution levels than playing a mixed strategy over this set of contribution levels. The existence of pure Nash equilibria is guaranteed by the standard fixed point argument since the payoff function is concave and the strategy set is compact and convex [13].

At the equilibria of this game, the strategy x_i^{ne} of user i should satisfy the following conditions,

$$\begin{cases} x_i^{ne} = w - \delta \sum_{j\in\mathcal{N}_i} t_{ij}x_j^{ne}, & \text{if } w > \delta \sum_{j\in\mathcal{N}_i} t_{ij}x_j^{ne}, \\ x_i^{ne} = 0, & \text{if } w \leq \delta \sum_{j\in\mathcal{N}_i i} t_{ij}x_j^{ne}. \end{cases} \quad (2)$$

where w is the point that $b'(w) = c$. From equation (2) we can see that a user will make up any shortfall from its neighbors' contributions to reach w, and exerts no contribution if the weighted sum of its neighbors' contributions has reached w. For small values of δ, Theorem I in [14] implies that if $\delta < 1/(1 + \rho(U - I - T))$, where U is a matrix whose entries are all 1s and I is the identity matrix, then there exists a unique interior solution for (2). Denote the unique solution by $\mathbf{x}^{ne}$. Then since $\mathbf{x}^{ne}$ is interior, according to equation (2) we have $(I + \delta T) \cdot \mathbf{x}^{ne} = w \cdot \mathbf{1}$. Due to the existence and uniqueness of $\mathbf{x}^{ne}$, we know that $(I + \delta T)$ is invertible and $\mathbf{x}^{ne} = w \cdot (I + \delta T)^{-1} \cdot \mathbf{1} = w \cdot (\mathbf{1} - \delta \cdot \mathbf{h}(-\delta, T))$. Therefore, in the Nash equilibrium nodes that have higher Bonacich centrality, i.e., with a higher value of $h_i(-\delta, T)$, exert less contributions. To simplify the notation, we use $\mathbf{h}$ to represent $\mathbf{h}(-\delta, T)$ in the rest of the paper.

Next we analyze the social optimal point of the game, which is the network operating point that all users are subject to centralized control and the social welfare achieves its maximum value. We define the social welfare to be the sum of all individual payoffs, i.e.,

$$SW(\mathbf{x}) = \sum_{i=1}^{n} [b(x_i + \delta \cdot \sum_{j\in\mathcal{N}_i} t_{ij}x_j) - c \cdot x_i], \ x_i \geq 0. \quad (3)$$

Since $SW(\mathbf{x})$ is concave, and the inequality constraints are convex, the Kuhn-Tucker conditions are the necessary and sufficient conditions for optimality. Therefore, the social optimum

$\mathbf{x}^{so}$ satisfies the following conditions,

$$\begin{cases} \text{if } x_i^{so} > 0,\ b'(x_i^{so} + \delta \sum_{j \in \mathcal{N}_i} t_{ij} x_j^{so}) + \\ \delta \sum_{j \neq i} t_{ji} b'(x_j^{so} + \delta \sum_{k \in \mathcal{N}_j} t_{jk} x_k^{so}) - c = 0, \\ \text{if } x_i^{so} = 0,\ b'(x_i^{so} + \delta \sum_{j \in \mathcal{N}_i} t_{ij} x_j^{so}) + \\ \delta \sum_{j \neq i} t_{ji} b'(x_j^{so} + \delta \sum_{k \in \mathcal{N}_j} t_{jk} x_k^{so}) - c \leq 0. \end{cases}$$

The social optimum $\mathbf{x}^{so}$ has the following property.

Lemma 4.1: Let $\mathbf{v} = [v_1, \ldots, v_n]$ be the vector that satisfies $[b'(v_1), \ldots, b'(v_n)]^T = c \cdot (I + \delta \cdot T)^{-1} \cdot \mathbf{1} = c \cdot (\mathbf{1} - \delta \cdot \mathbf{h})$, and define $\mathbf{x}^* = (I + \delta T)^{-1} \cdot \mathbf{v}$. If $\mathbf{x}^*$ is non-negative, we have $\mathbf{x}^{so} = \mathbf{x}^*$. Otherwise, $\mathbf{x}^{so}$ is different from $\mathbf{x}^*$ but we have $SW(\mathbf{x}^{so}) < SW(\mathbf{x}^*)$.

Proof: By simple manipulation of liner algebra, we know that $\mathbf{x}^*$ is the solution for the following problem

$$\max_{\mathbf{x}} \sum_{i=1}^{n} [b(x_i + \delta \sum_{j \neq i} t_{ij} x_j) - c \cdot x_i], \tag{4}$$

which is only different from the maximization problem in (3) by dropping the non-negative constraints on x_i's; so if $\mathbf{x}^*$ is non-negative, it is also the optimal solution for (3). Otherwise, $\mathbf{x}^*$ is not the optimal solution for (3), but the maximal objective value of (4) is never smaller than that of (3), so we have $SW(\mathbf{x}^{so}) < SW(\mathbf{x}^*)$ ■

From the above analysis, we can see that both the Nash equilibrium and the social optimum of this game is closely related to the Bonacich centralities of the nodes in the trust-weighted network. To describe the difference in maximum social welfare $SW(\mathbf{x}^{so})$ and the social welfare at the Nash equilibrium $SW(\mathbf{x}^{ne})$, we compute the *Price of Anarchy* (PoA) for this game. The PoA measures the price that the users will pay if they play selfishly rather than play according to a centralized control. It is defined as the ratio of the social optimal welfare to the welfare of the worst Nash equilibrium. Since the Nash equilibrium of this game is unique when $\delta < 1/(1+\rho(U-I-T))$, and we have $SW(\mathbf{x}^{so}) \leq SW(\mathbf{x}^*)$, the PoA of this game can be upper bounded by

$$PoA \leq \frac{SW(\mathbf{x}^*)}{SW(\mathbf{x}^{ne})} = \frac{\sum_{i=1}^{n} [b(x_i^* + \delta \sum_{j \in \mathcal{N}_i} t_{ij} x_j^*) - c \cdot x_i^*]}{\sum_{i=1}^{n} [b(x_i^{ne} + \delta \sum_{j \in \mathcal{N}_i} t_{ij} x_j^{ne}) - c \cdot x_i^{ne}]}.$$

Lemma 4.2: The PoA of this game can be upper bounded by $PoA \leq \sum_i h_i \cdot v_i / \sum_i h_i \cdot w$.

Proof: Since $b(\cdot)$ is a concave function, $b'(w) = c$, $(I + \delta \cdot T) \cdot \mathbf{x}^* = \mathbf{v}$ and $(I + \delta \cdot T) \cdot \mathbf{x}^{ne} = \mathbf{w}$, we have

$$\begin{aligned} & b(x_i^* + \delta \sum_{j \in \mathcal{N}_i} t_{ij} x_j^*) - b(x_i^{ne} + \delta \sum_{j \in \mathcal{N}_i} t_{ij} x_j^{ne}) \\ = \ & b(v_i) - b(w) \leq (v_i - w) \cdot b'(w) = (v_i - w) \cdot c. \end{aligned}$$

Let $\mathbf{c} = c \cdot \mathbf{1}$ and $\mathbf{w} = w \cdot \mathbf{1}$, then the PoA can be written as,

$$\begin{aligned} PoA \ & \leq \ \frac{\sum_{i=1}^{n} [b(w) + c \cdot (v_i - w) - c \cdot x_i^*]}{\sum_{i=1}^{n} [b(w) - c \cdot x_i^{ne}]} \\ & = \ 1 + \frac{\sum_{i=1}^{n} [c \cdot (v_i - w) - c \cdot (x_i^* - x_i^{ne})]}{\sum_{i=1}^{n} [b(w) - c \cdot x_i^{ne}]} \\ & < \ 1 + \frac{\mathbf{c}^T (\mathbf{v} - \mathbf{w}) - \mathbf{c}^T (I + \delta T)^{-1} (\mathbf{v} - \mathbf{w})}{n \cdot w \cdot b'(w) - \mathbf{c}^T (I + \delta T)^{-1} \mathbf{w}} \\ & = \ 1 + \frac{\delta \mathbf{c}^T (I - (I + \delta T)^{-1})(\mathbf{v} - \mathbf{w})}{\mathbf{c}^T (I - (I + \delta T)^{-1}) \mathbf{w}} \\ & = \ 1 + \frac{\delta \mathbf{c}^T T (I + \delta T)^{-1} (\mathbf{v} - \mathbf{w})}{\delta \mathbf{c}^T T (I + \delta T)^{-1} \mathbf{w}} \\ & = \ \frac{\mathbf{1}^T \cdot T (I + \delta T)^{-1} \mathbf{v}}{\mathbf{1}^T \cdot T (I + \delta T)^{-1} \mathbf{w}} = \frac{\sum_{i=1}^{n} h_i \cdot v_i}{\sum_{i=1}^{n} h_i \cdot w} \end{aligned} \tag{5}$$

■

Lemma 4.2 shows that the upper bound of the PoA depends on the Bonacich centralities of nodes in the trust-weighted network. Since w is always smaller than v_i and a node with higher Bonacich centrality has a higher value of v_i, the larger variance the Bonacich centralities, the lower the upper bound of game efficiency. An intuitive explanation for the low efficiency is that although the contributions from users with higher Bonacich centrality would benefit more users compared to the contributions from users with lower centrality, those users tend to exert less contribution in the Nash equilibrium because they can rely more on their neighbors.

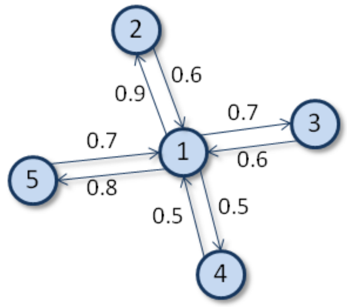
V. Heterogeneity and Efficiency Improvement

To improve game efficiency, one way is to increase the contributions from the nodes that have higher Bonacich centralities. Observing that nodes who have more resources will have higher willingness to contribute, we propose to introduce more resources to those nodes that have higher Bonacich centralities, so they may contribute more at the Nash equilibrium.

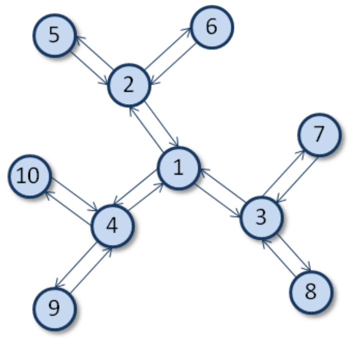
We model the different resources in nodes by a discount factor on cost, i.e., after introducing more resources, the cost for user i to make contribution x_i is now $(1-\gamma_i) \cdot c \cdot x_i$, where $\gamma_i \in [0, 1)$ depends on the amount of resources introduced. If a node has more resources, the cost for it to make the same level of contributions will be smaller than nodes with fewer resources. There are two important issues that need to be addressed, i.e., the amount and the position of heterogeneous resource to introduce. We first study two example networks in Figure 1a and 1b, and then provide mathematical analysis for these two issues.

The links in Figure 1a and 1b represent the trust relations among nodes. In Figure 1a trust values are displayed with the links. In Figure 1b we assume all the trust values are 1, in order to focus on the underlying structure of the network. We use benefit function $b(x) = -x^2 + 2x, x \in [0, 1)$ and let the cost coefficient be $c = 1$. The benefit function is chosen to satisfy the concavity constraint. The parameter δ is set to 0.2 for Figure 1a and 0.1 for 1b, in order to satisfy the constraint $\delta < 1/(1 + \rho(U - I - T))$.

For Figure 1a, the Bonacich centrality measurement for the nodes is $\mathbf{h} = [2.7400, 0.2712, 0.2712, 0.2260, 0.3164]^T$.

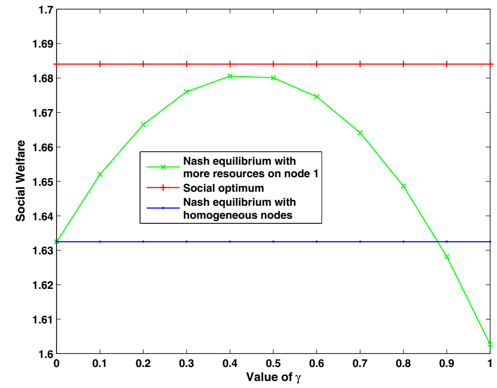


(a) Network I

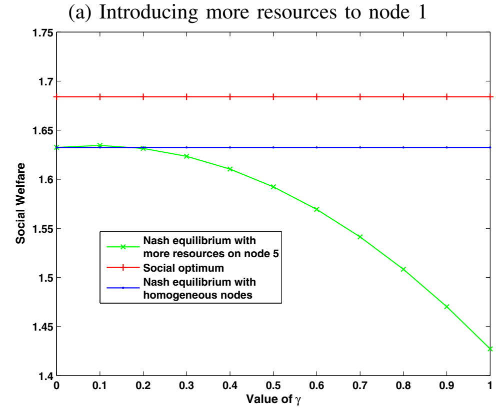


(b) Network II

Fig. 1: Two Example Networks



(a) Introducing more resources to node 1



(b) Introducing more resources to node 5

Fig. 2: Efficiency Improvement for Network I

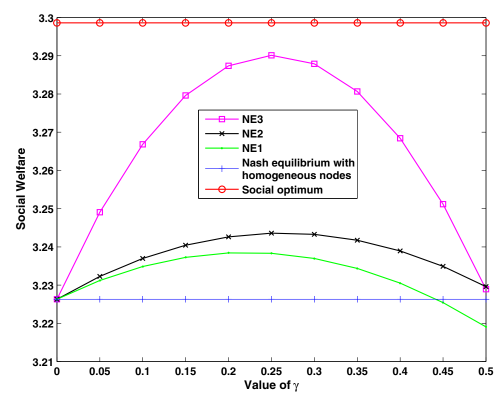


Fig. 3: Efficiency Improvement for Network II

The Nash equilibrium of the game is achieved at $\mathbf{x}^{ne} = [0.2260, 0.4729, 0.4729, 0.4774, 0.4684]^T$. As expected, the center node, which has much higher Bonacich centrality than others, contributes much less in the Nash equilibrium. However, the social optimum is obtained at $\mathbf{x}^{so} = [0.5037, 0.4667, 0.4667, 0.4722, 0.4611]^T$, in which the center node makes the highest contributions. To improve game efficiency, we introduce heterogeneous resources for the nodes. Figure 2a and Figure 2b illustrate the social welfare at the various Nash equilibria when introducing more resources to node 1 and 5. Obviously, the social welfare can be improved when node 1 is supplied by an appropriate level of additional resources. But introducing more resources to node 5 actually reduces the social welfare. Therefore, we should design carefully where to introduce the heterogeneous resources. Also, it is observed in Figure 2a that the social welfare does not monotonically increase with the increasing of resources at the center node, which poses another challenge for the design, i.e., to determine the optimal amount of additional resources.

For the network in Figure 1b, although node 1 appears to be the most 'central' in the network, it actually has lower Bonacich centrality than node 2, 3 and 4. This is because Bonacich centrality puts negative weights on even length paths. The actual Bonacich centrality is 2.2105 for node 1, 2.6316 for node 2, 3 and 4, and 0.7368 for node 5, 6, 7, 8, 9, 10. Figure 3 shows the social welfare improvement by introducing more resources to node 1 (NE1), node 2 (NE2) and node 1, 2, 3, 4 together (NE3). We observe that the game efficiency is not improved much by introducing more resources for node 1 or node 2 only, but is significantly improved by introducing more resources for node 1, 2, 3 and 4 together. That is, not only where the additional resources should be introduced but also how many nodes should be allocated with additional resources need to be carefully designed.

Now we mathematically analyze the possible improvement of game efficiency by introducing heterogeneous resources. Let $\tilde{\mathbf{c}} = (1 - \boldsymbol{\gamma}) \cdot c$ be the cost for users to make a unit effort after we introduce additional resources to them, where $\boldsymbol{\gamma} = [\gamma_1, \ldots, \gamma_n]$ is the vector of cost discount factors for users. Assuming $\tilde{\mathbf{w}}$ is the vector that satisfies $[b'(\tilde{w}_1), \ldots, b'(\tilde{w}_n)]^T = \tilde{\mathbf{c}}$, note that due to the concavity of $b(\cdot)$, the value of $\tilde{w}_i$ increases with the increase of γ_i , the strategy for user i at the new Nash Equilibrium, denoted by $\tilde{\mathbf{x}}^{ne}$, becomes

$$\begin{cases} \tilde{x}_i^{ne} = \tilde{w}_i - \delta \sum_{j \in \mathcal{N}_i i} t_{ij} \tilde{x}_j{}^{ne} & \text{if } \delta \sum_{j \in \mathcal{N}_i} t_{ij} \tilde{x}_j{}^{ne} < \tilde{w}_i, \\ \tilde{x}_i^{ne} = 0 & \text{if } \delta \sum_{j \in \mathcal{N}_i} t_{ij} \tilde{x}_j{}^{ne} \geq \tilde{w}_i. \end{cases} \quad (6)$$

Since equation (6) is also the Kuhn-Tucker condition for the following maximization problem

$$\max_{\mathbf{x}} \ \mathbf{x}^T \cdot \tilde{\mathbf{w}} - \frac{1}{2} \mathbf{x}^T (I + \delta T) \mathbf{x}, \forall\ i,\ x_i \geq 0, \quad (7)$$

and $(I+\delta T)$ is invertible, there is a unique solution to problem (7) and thus a unique Nash equilibrium for the new game. Furthermore, if $(I+\delta T)^{-1}\tilde{\mathbf{w}}$ is non-negative, we have $\tilde{\mathbf{x}}^{ne} = (I+\delta T)^{-1}\tilde{\mathbf{w}}$. Otherwise, the Nash equilibrium will have some i with $\tilde{x}_i^{ne} = 0$. The non-negativeness of $(I+\delta T)^{-1}\tilde{\mathbf{w}}$ depends on the choice of $\tilde{\mathbf{c}}$ and the benefit function $b(\cdot)$.

Lemma 5.1: When $(I+\delta T)^{-1}\cdot\tilde{\mathbf{w}}$ is non-negative, thus being the Nash equilibrium for the new game, the improvement of social welfare at the Nash equilibrium can be bounded by

$$SW(\tilde{\mathbf{x}}^{ne}) - SW(\mathbf{x}^{ne}) \geq \sum_{\{i:\gamma_i>0\}} (\tilde{w}_i - w)(\delta\cdot h_i - \gamma_i)\cdot c \quad (8)$$

$$SW(\tilde{\mathbf{x}}^{ne}) - SW(\mathbf{x}^{ne}) \leq \sum_{\{i:\gamma_i>0\}} \delta\cdot c\cdot(\tilde{w}_i - w)\cdot h_i, \quad (9)$$

and the improvement on the PoA value can be bounded by

$$\frac{SW(\tilde{\mathbf{x}}^{ne})}{SW(\mathbf{x}^{ne})} \leq \frac{\sum_i \tilde{w}_i h_i}{\sum_i w h_i}. \quad (10)$$

Proof: Since $b(\cdot)$ is concave, $(I+\delta T)\cdot\tilde{\mathbf{x}}^{ne} = \tilde{\mathbf{w}}$, and $(I+\delta T)\cdot\mathbf{x}^{ne} = \mathbf{w}$, the improvement of social welfare at the equilibrium by introducing heterogeneity can be written as

$$\begin{aligned}
\tilde{SW}^{ne} - SW^{ne} &= \sum_{i=1}^{n}[b(\tilde{w}_i) - b(w) + c(x_i^{ne} - \tilde{x}^{ne})]\\
\tilde{SW}^{ne} - SW^{ne} &\geq \tilde{\mathbf{c}}^T(\tilde{\mathbf{w}} - \mathbf{w}) + \mathbf{c}^T(\mathbf{x}^{ne} - \tilde{\mathbf{x}}^{ne})\\
&= (\tilde{\mathbf{w}} - \mathbf{w})^T(\tilde{\mathbf{c}} - (I+\delta T)^{-1}\mathbf{c})\\
&= \sum_{i=1}^{n}(\tilde{w}_i - w)(\delta\cdot h_i - \gamma_i)\cdot c,\\
\tilde{SW}^{ne} - SW^{ne} &\leq \mathbf{c}^T(\tilde{\mathbf{w}} - \mathbf{w}) + \mathbf{c}^T(\mathbf{x}^{ne} - \tilde{\mathbf{x}}^{ne})\\
&= (\tilde{\mathbf{w}} - \mathbf{w})^T(\delta T(I+\delta T)^{-1}\mathbf{c})\\
&= \sum_{i=1}^{n}\delta\cdot c\cdot(\tilde{w}_i - w)\cdot h_i.
\end{aligned}$$

Following a similar procedure as in the proof of Lemma 4.2, we have

$$\frac{SW(\tilde{\mathbf{x}}^{ne})}{SW(\mathbf{x}^{ne})} \leq \frac{\delta\tilde{\mathbf{w}}^T T(I+\delta T)^{-1}\cdot\mathbf{c}}{\delta\mathbf{w}^T T(I+\delta T)^{-1}\mathbf{w}} = \frac{\sum_i \tilde{w}_i h_i}{\sum_i w h_i}.$$

■

The upper bound on the social welfare improvement in (9) is monotonically increasing with the increase of cost discount factor. But from the lower bound in equation (8), we can see that increasing the cost discount factor to the nodes that have higher Bonacich centralities could increase the value of the term $(\tilde{w}_i - w)$ but decrease the value of the term $(\delta\cdot h_i - \gamma_i)$, so the social welfare improvement may not monotonically increase, which is consistent with the observation in Figure 2a. Also, by equation (10) we can expect that the higher the variance of the Bonacich centralities of nodes in the trust-weighted network, the more room we have to improve game efficiency by introducing heterogeneous resources.

When the non-negativeness of $(I+\delta T)^{-1}\tilde{\mathbf{w}}$ is not satisfied, the Nash equilibrium of the new game will have some i such that $\tilde{x}_i^{ne} = 0$, which is a much more involved case. We leave it as our future work.

VI. Conclusions and Future Work

In this paper, we study the effects of trust relations among users on the equilibria in network games. We consider a game in which each user locally interacts with its neighbors, where the local interactions are affected by their trust relations. We characterize the unique Nash equilibrium in this game using Bonacich centrality measures, and prove that the game efficiency, measured in terms of the price of anarchy, is also highly related to this centrality measure. The more spread the Bonacich centralities among nodes, the lower the game efficiency. Motivated by the observation that users that have more resources will have higher willingness to contribute, we propose to improve game efficiency by introducing heterogeneous resources according to nodes' centralities. We provide both experimental and theoretical analysis on the efficiency improvement by introducing heterogeneity.

In our future work, we will extend this static game to repeated rounds to capture the evolution of trust and cooperation among nodes, and explore the potential of heterogeneity on improving Nash equilibrium quality when the trust relations change dynamically.

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